

# Extracting Muon Momentum Scale Corrections for Hadron Collider Experiments

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**Abstract.** We present a simple method for the extraction of corrections for bias in the measurement of the momentum of muons in hadron collider experiments. Such bias can originate from a variety of sources such as detector misalignment, software reconstruction bias, and uncertainties in the magnetic field. The two step method uses the mean  $\langle 1/p_T^\mu \rangle$  for muons from  $Z \rightarrow \mu\mu$  decays to determine the momentum scale corrections in bins of charge,  $\eta$  and  $\phi$ . In the second step, the corrections are tuned by using the average invariant mass  $\langle M_{\mu\mu}^Z \rangle$  of  $Z \rightarrow \mu\mu$  events in the same bins of charge  $\eta$  and  $\phi$ . The forward-backward asymmetry of  $Z/\gamma^* \rightarrow \mu\mu$  pairs as a function of  $\mu^+\mu^-$  mass, and the  $\phi$  distribution of  $Z$  bosons in the Collins-Soper frame are used to ascertain that the corrections remove the bias in the momentum measurements for positive versus negatively charged muons. By taking the sum and difference of the momentum scale corrections for positive and negative muons, we isolate additive corrections to  $1/p_T^\mu$  that may originate from misalignments and multiplicative corrections that may originate from mis-modeling of the magnetic field ( $\int \mathbf{B} \cdot d\mathbf{L}$ ). This method has recently been used in the CDF experiment at Fermilab and in the CMS experiment at the Large Hadron Collider at CERN.

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## 1 Introduction

In general, the reconstruction of the momentum of muons in hadron collider experiments (e.g., CDF, D0, ATLAS, CMS) is biased. Bias originates from detector misalignments, the reconstruction software, and uncertainties in the magnetic field ( $\int \mathbf{B} \cdot d\mathbf{L}$ ). Monte Carlo (MC) generated events start with no biases, but inaccurate inputs for the detector alignment, magnetic field, and running conditions can induce biases during the reconstruction of the events. The bias in the reconstructed momentum of muons depends on the charge of the muon, and on the  $\eta$  and  $\phi$  coordinates[1] of the muon track. The bias in the reconstruction of the muon momentum in the data and in the simulated events is not necessarily the same. Therefore, comparison of data and reconstructed MC events require the removal of the bias from both data and reconstructed MC samples.

Precision measurements such as the charge asymmetry in the production of  $W$  bosons, the measurement of the forward-backward asymmetry ( $A_{fb}$ ) of  $Z/\gamma^* \rightarrow \mu\mu$  events as a function of the  $\mu^+\mu^-$  mass, measurements of angular distributions, and searches for new high mass states decaying to  $\mu^+\mu^-$  pairs are very sensitive to bias in the measurement of muon momenta. The reconstruction bias also worsens the detector resolution since it depends on the charge of the muon and the  $\eta$  and  $\phi$  coordinates of the muon track. In this paper, we present a simple data-driven

method for the extraction of misalignment and muon scale corrections from the  $Z/\gamma^* \rightarrow \mu\mu$  event samples. The paper is organized as follows. In section 1.1 we present a general overall view of the method. We then follow with additional details and application of the method to real collider data.

### 1.1 The Method

Since the  $Z$  mass is well known,  $Z/\gamma^* \rightarrow \mu\mu$  events have been previously used to check on the momentum scale of reconstructed muons. The difficulty is that the  $\mu^+$  and  $\mu^-$  are correlated, and the mass of the final state depends on the momentum of the two muons. Therefore, we devise a two step process as described below.

In the first step, we obtain initial corrections in bins of charge,  $\eta$  and  $\phi$ . These corrections are uncorrelated, remove all the bias, and yield the correct average mass of the  $Z$  boson for the sample. In the second step we fine-tune the corrections using the mass of the  $Z$  boson for each bin of charge ( $Q$ ),  $\eta$  and  $\phi$ .

#### 1.1.1 Monte Carlo sample for a perfectly aligned detector

We begin by constructing a MC sample of  $Z/\gamma^* \rightarrow \mu\mu$  events for a perfectly aligned and unbiased detector as

follows. We start with a simulated sample of  $Z/\gamma^* \rightarrow \mu\mu$  events with identical selection cuts as the data. We know that the reconstruction of the momentum for these MC events may be biased. Therefore, instead of using the reconstructed MC information, we use the generated momentum and smear it with a functional form that represents the experimental resolution as function of  $\eta$ . This process yields a sample of  $Z/\gamma^* \rightarrow \mu\mu$  events for a perfectly aligned detector.

For the Tevatron, we use PYTHIA [2] for the generated sample and weight the transverse momentum and rapidity distributions [1] by correction factors to bring the distributions into agreement with published[3] CDF data.

For the LHC, we use POWHEG [4] for the generated sample and weight the transverse momentum and rapidity distributions of the  $Z$  bosons by a correction factor to bring them into agreement with published[5] CMS data.

### 1.1.2 Summary of the first step

In the first step, we obtain initial momentum scale corrections in bins of charge ( $Q$ ),  $\eta$  and  $\phi$  by requiring that the average of  $1/p_T^\mu$  ( $\langle 1/p_T^\mu \rangle$ ) of selected muons from  $Z$  decays for data and reconstructed MC to be the same as that for the perfectly aligned sample. This yields a lookup table of momentum scale corrections for  $\mu^+$  and  $\mu^-$  events (separately) for both data and reconstructed MC. Since we use a lookup table, we are not constrained by a particular functional form for the parametrization of the corrections. These corrections remove all bias in the reconstructed momentum.

We refer to this step as the  $\langle 1/p_T^\mu \rangle$  based corrections. Since we only use the  $\langle 1/p_T^\mu \rangle$  for individual muons, the correction for each bin in  $\eta/\phi$  is uncorrelated with the correction for any of the other  $\eta/\phi$  bins. Since the corrected  $\langle 1/p_T^\mu \rangle$  for each bin is now the same as the  $\langle 1/p_T^\mu \rangle$  for the perfectly aligned and unbiased MC sample, the average mass of the  $Z/\gamma^* \rightarrow \mu\mu$  is also correct. When this procedure is applied to the sample of reconstructed MC events, we find that these first step corrections remove all the biases in the reconstructed momentum for all bins in  $Q$ ,  $\eta$  and  $\phi$ .

### 1.1.3 Summary of the second step

When we extract these first step corrections for the data, we assume that the perfectly aligned MC sample correctly models the rapidity and transverse momentum distributions for the production and decay of  $Z/\gamma^* \rightarrow \mu\mu$  events, including final state radiation of photons. In addition, we assume that the MC correctly models the detector acceptance and efficiencies. These assumptions are correct on average because the rapidity and transverse momentum distributions for the MC is usually tuned to describe the data. Similarly, the efficiency for the reconstruction of muons as a function of the muon  $\eta$  is also extracted from the data. Nonetheless, when we apply this procedure to the data, we find that although the average mass

of  $Z \rightarrow \mu^+\mu^-$  events is correct, we see random scatter in the average  $Z$  mass for different  $\eta/\phi$  bins in data which is not seen in the corrected reconstructed MC sample. This random scatter is due to the fact that there are variations in the muon detection efficiency for the different  $\eta/\phi$  bins, which is not perfectly modeled in the MC.

In order to be independent of modeling of detector efficiencies, and also be independent of the modeling assumptions for the production of  $Z/\gamma^* \rightarrow \mu\mu$  events as a function of rapidity and transverse momentum, we fine-tune the corrections by requiring that the reconstructed  $Z$  mass is the same as for the perfectly aligned detector for  $\mu^+$  and  $\mu^-$  events in each bin in  $\eta$  and  $\phi$ . This removes the scatter in the average  $Z$  mass for different  $\eta/\phi$  bins. We refer to this step as the  $\Delta M/M$  tuning.

Next, by taking the sum and difference of the  $\mu^+$  and  $\mu^-$  momentum scale corrections, we extract *additive* (in  $1/p_T^\mu$ ) corrections that are caused by misalignments, and *multiplicative* (in  $1/p_T^\mu$ ) corrections that are caused by mis-modeling of the magnetic field (or from mis-modeling of the integral of  $\mathbf{B} \cdot d\mathbf{L}$ ) as a function of  $\eta$  and  $\phi$ .

A detailed description of the method is given below.

## 2 Data Set and Event Selection

For CMS, we use  $Z \rightarrow \mu\mu$  events generated by the POWHEG Monte Carlo[4] followed by PYTHIA which models parton showering and final state radiation. We apply event weighting corrections which are a function of  $Z$  transverse momentum ( $P_T^Z$ ) and rapidity ( $y$ ) to ensure that the transverse momentum and rapidity distributions in MC match the data.

For both data and reconstructed MC events we require both muons to be isolated. In the definition of isolation for a muon we use information only from the track and hadron calorimeter. If the electromagnetic (EM) calorimeter energy is not included in the isolation requirement, the momentum dependence of the efficiency is expected to be constant. If the EM energy is included in the isolation requirement, then photons from final state radiation result in a momentum dependence of the efficiency, and also in a complicated correlation between the efficiency of the two muons.

For example, for the CMS detector we use the following selection requirements:

- $p_T^\mu > 25$  GeV/c on the muon with the largest  $p_T$  to ensure high muon trigger efficiency, and  $p_T^\mu > 20$  GeV/c for the second muon.
- Detector  $|\eta| < 2.4$  (tracker acceptance)
- Mass selection:  $60 < M_{\mu\mu} < 120$  GeV/ $c^2$  for the  $1/p_T$  based correction
- $86.5 < M_{\mu\mu} < 96.5$  GeV/ $c^2$  for cross checks on the  $\Delta M/M$  tuning.

### 3 Reference Plots Used in the Muon Momentum Study

A misalignment of the tracker generates distortions in several kinematic distributions of Drell-Yan ( $Z/\gamma^* \rightarrow \mu\mu$ ) events in the  $Z$  boson mass region.

In our analysis we use several kinematic distributions, including the invariant mass of the dimuon pair ( $M_{\mu\mu}$ ), the angles  $\theta_{CS}$  and  $\phi_{CS}$  of the negatively charged muon in the Collins-Soper frame [6], and the forward-backward asymmetry of the negatively charged muon in the Collins-Soper frame.

The Collins-Soper frame is the rest frame of the dilepton pair. In this frame, we define the momentum vector of the beam particle as  $\mathbf{P}_A$  and the momentum vector of the target particle as  $\mathbf{P}_B$ . For proton-antiproton collisions (e.g. Tevatron) the beam particle is defined as the proton and the target particle is defined as the antiproton. The  $z$ -axis bisects the beam particle direction and the opposite of the target particle direction in the dilepton rest frame. The positive  $z$  axis is along the beam particle direction. For proton-proton collisions (e.g. LHC), the beam particle (i.e. positive  $z$  axis) is defined as the proton beam that points in the direction of the rapidity of the dilepton pair.

The angles  $\theta_{CS}$  and  $\phi_{CS}$  are defined [6] by

$$\begin{aligned} \cos \theta_{CS} &= \frac{2}{M_{\mu\mu} \sqrt{M_{\mu\mu}^2 + P_T^2}} (p_1^+ p_2^- - p_1^- p_2^+) \\ \tan \phi_{CS} &= \frac{\sqrt{M_{\mu\mu}^2 + P_T^2}}{M_{\mu\mu}} \cdot \frac{\Delta_r \cdot \hat{R}_T}{\Delta_r \cdot \hat{P}_T} \end{aligned} \quad (1)$$

Here,  $p_1$  and  $p_2$  are the four-momentum of negatively and positively charged muons, respectively,  $P_T$  is the transverse momentum of the dimuon pair in the laboratory system,  $p^\pm$  corresponds to  $\frac{1}{\sqrt{2}}(p^0 \pm p^3)$ ,  $\Delta^j = p_1^j - p_2^j$ ,  $\hat{P}_T$  is a transverse unit vector in the direction of  $\mathbf{P}_T$ , and  $\hat{R}_T$  is a transverse unit vector in the direction of  $\mathbf{P}_A \times \mathbf{P}_B$ .

We define  $\phi_{CS}$  to be the angle between the direction of the  $Z/\gamma^*$  boson  $p_T$  and the direction of the negatively charged lepton,

When integrated over all  $\phi_{cs}$  the differential cross section can be written as :

$$\frac{d\sigma}{d\cos\theta} \propto (1 + \cos^2\theta) + \frac{1}{2}A_0(1 - 3\cos^2\theta) + A_4\cos\theta \quad (2)$$

where  $A_0(M_{\mu\mu}, y, p_T)$  originates from QCD gluon radiation and  $A_4(M_{\mu\mu}, y, p_T)$  originates from electroweak interference. The forward (f) backward (b) asymmetry in the Collins-Soper frame is defined as.

$$A_{fb} = \frac{N_f - N_b}{N_f + N_b} \quad (3)$$

where  $N_f$  and  $N_b$  are the number of events for positive and negative  $\cos\theta_{CS}$ , respectively. For a detector with 100% acceptance over all  $\cos\theta_{CS}$  the forward-backward asymmetry is given by  $A_{fb} = \frac{3}{8}A_4$ .

Since the misalignments in data and MC are different, the distributions are distorted in different ways for data and MC. Detector misalignments may be responsible for the following:

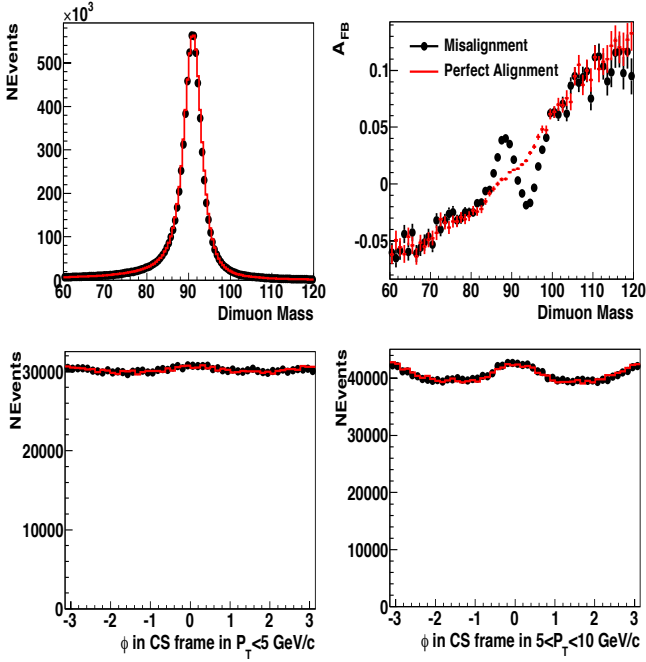
- Cause a charge,  $\eta$ , and  $\phi$  dependent bias in the measurement of the muon momentum which also worsens the resolution.
- Cause a charge,  $\eta$ , and  $\phi$  dependence of the average reconstructed  $Z$  boson mass.
- Distort and widen the overall shape of the  $Z/\gamma^* \rightarrow \mu\mu$  mass distributions.
- A charge dependence in the reconstructed muon momentum creates unphysical wiggles in the forward and backward lepton angle asymmetry ( $A_{fb}$ ) in the Collins-Soper [6] (CS) dilepton rest frame for Drell-Yan events as a function of dilepton mass (in the region of the  $Z$  peak). This yields one of two powerful checks on a difference in the momentum scale between positive and negative muons.
- For low  $p_T$   $Z/\gamma^*$  bosons ( $p_T^Z < 10$  GeV/c), the  $\phi$  distribution in the CS frame ( $\phi_{CS}$ ) is expected to be flat. However, since we define  $\phi_{CS}$  to be the angle between the direction of the  $Z/\gamma^*$  boson  $p_T$  and the direction of the negatively charged lepton, resolution smearing in the measurement of the muon momentum results in an excess of events near  $\phi_{CS} = 0$  and  $\pm\pi$  in the reconstructed  $\phi_{CS}$  distribution. The level of the excess at  $\phi_{CS} = 0$  and  $\pm\pi$  is expected to be the same if the muon momentum scales and resolutions are the same between  $\mu^+$  and  $\mu^-$ . For  $Z/\gamma^*$  events with  $p_T^Z = 0$  there is no preferred  $x$  axis. However, if there is a difference in the reconstruction bias for positive and negative muons, events which are produced with  $p_T^Z = 0$  are reconstructed with  $p_T^Z$  along either the positive ( $\phi_{CS} = 0$ ) or the negative muon ( $\phi_{CS} = \pm\pi$ ) direction. Therefore, the  $\phi_{CS}$  distribution in the low  $p_T^Z$  region provides the second powerful check on a difference in the momentum scale between positive and negative muons.

In our study we use the following two kinematic distributions as reference plots to test the validity of the momentum corrections. These reference plots are not used in the extraction of the momentum corrections. They are only used to ascertain that the correction factors actually work.

- $A_{fb}$  for  $Z/\gamma^* \rightarrow \mu\mu$  events as a function of mass.
- $\phi_{CS}$  in two  $Z/\gamma^*$   $p_T$  bins:  $0 < p_T^Z < 5$  GeV/c, and  $5 < p_T^Z < 10$  GeV/c.

The following distributions are used to determine the momentum correction factors and also determine the  $\eta/\phi$  dependence of the momentum resolution:

- The  $1/p_T^\mu$  distributions for positive and negative muons in bins of  $\eta$  and  $\phi$ .
- The overall dimuon invariant mass spectrum ( $M_{\mu^+\mu^-}$ ).
- The average  $Z/\gamma^*$  mass in the  $Z$  peak region as a function of  $\eta$  and  $\phi$  of the  $\mu^+$  or the  $\mu^-$ . If the  $\mu^+$  of the pair is binned in  $\eta$  and  $\phi$ , its partner is allowed to be

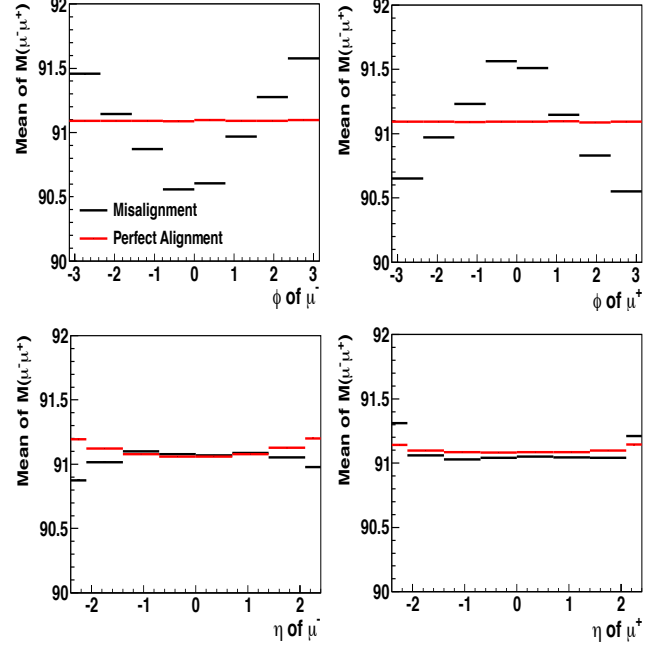


**Fig. 1.** An example of the first set of reference plots ( $M_{\mu^+\mu^-}$ ,  $A_{fb}$ , and  $\phi_{CS}$ ) for a CMS-like detector at the LHC for 7 TeV in the center of mass. The red histograms are the distributions for a perfectly aligned detector and the black points are for one example of a misaligned detector. The kinematic selection cuts are: muon  $p_T^\mu > 20$  GeV/c and  $|\eta| < 2.4$  for both muons and  $60 < M_{\mu\mu} < 120$  GeV/c<sup>2</sup>. Top Plots: The  $\mu^+\mu^-$  invariant mass distribution (left) and  $A_{fb}$  (right). Bottom plots: The  $\phi$  distribution in the Collins-Soper frame in boson  $p_T^Z < 5$  GeV/c (left) and  $\phi$  in the Collins-Soper frame in boson  $5 < p_T^Z < 10$  GeV/c (right) distributions. (Color online).

in any bin, and vice versa. A broad window of 60–120 GeV/c<sup>2</sup> is used for initial tuning, and a tighter window of 86.5–96.5 GeV/c<sup>2</sup> is used for the final tuning.

We use the same procedure to extract the corrections for data and reconstructed MC. Since for the MC we know the generated muon momentum, we can use the generated information in the MC sample as an additional check on the procedure. Fig. 1 and 2 show examples of reference plots for a perfectly aligned MC for a CMS-like detector for proton-proton collisions at 7 TeV in the center of mass (red histograms). Also shown are the same reference plots for one example of a misaligned detector (black points). Only generated information was used to produce these sample reference plots. For purpose of illustration we have assumed 100% efficiency and a CMS-like momentum resolution. The following kinematic selection cuts were applied:  $60 < M_{\mu\mu} < 120$  GeV/c<sup>2</sup>, muon  $p_T^\mu > 20$  GeV/c and  $|\eta| < 2.4$  for both muons. In an actual application, the reference plots should also include the effect of detector efficiency and geometrical cuts which are specific to the experiment.

The reference plots which are shown in Fig. 1 and 2 are the  $M_{\mu\mu}$  distribution,  $A_{fb}$  versus  $M_{\mu\mu}$ , the distributions in  $\phi_{cs}$  for  $P_T^Z < 5$  GeV/c, and for  $5 < P_T^Z < 10$  GeV/c,



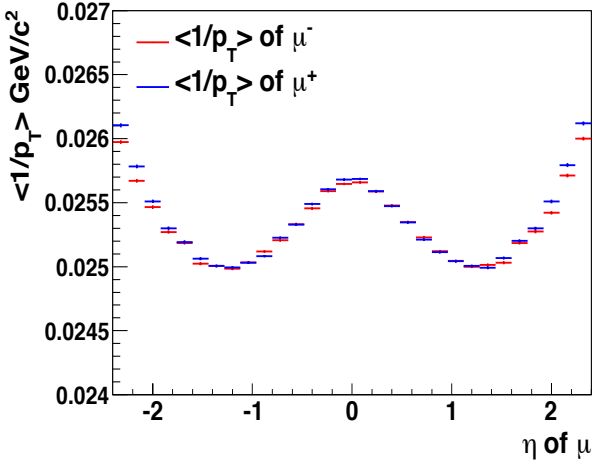
**Fig. 2.** An example of the second set of reference plots for a CMS-like detector at the LHC for 7 TeV in the center of mass. The red histograms are the distributions for a perfectly aligned detector and the black points are for one example of a misaligned detector. Shown are the reference plots for average  $Z$  mass ( $86.5 < M_{\mu\mu} < 96.5$  GeV/c<sup>2</sup>) as a function of  $\phi$  (top) or  $\eta$  (bottom) of the  $\mu^+$  and  $\mu^-$ . The kinematic selection cuts are: muon  $p_T > 20$  GeV/c and  $|\eta| < 2.4$  for both muons.

and the average  $Z$  mass ( $86.5 < M_{\mu\mu} < 96.5$  GeV/c<sup>2</sup>) versus muon  $\eta$  and  $\phi$  for positive and negative muons. (Note that the very small QCD, EW (diboson),  $\tau^+\tau^-$  and top-antitop background is also included in the distributions).

The reference plots for the reconstructed data (and reconstructed MC) should be in agreement with these perfect alignment reference plots after all the momentum scale corrections are applied.

#### 4 Muon Momentum Correction (step 1): $\langle 1/p_T^\mu \rangle$ based corrections

The correction factor  $C^{Data/MC}(Q, \eta, \phi)$ , is defined as the difference in the mean  $\langle 1/p_T^\mu \rangle$  between the mean  $\langle 1/p_T^\mu \rangle$  for an ideal perfectly aligned MC and reconstructed data (or reconstructed MC). Since the  $\langle 1/p_T^\mu \rangle$  based muon momentum correction factors are obtained in the range of  $p_T^\mu > 25$  GeV/c, the corrections are iterated to account for the fixed  $p_T^\mu > 25$  GeV/c until the mean  $\langle 1/p_T^\mu \rangle$  of muons in the corrected data (or corrected reconstructed MC) agree with the mean  $\langle 1/p_T^\mu \rangle$  of the perfectly aligned MC. Note that because of the  $p_T^\mu > 25$  GeV/c requirement, the mean  $\langle 1/p_T^\mu \rangle$  is a function of  $\eta$ , as shown in Fig. 3. At large  $\eta$ , because of electroweak interference, the mean  $\langle 1/p_T^\mu \rangle$  is different for positive muons which are



**Fig. 3.** The mean  $\langle 1/p_T^\mu \rangle$  for a perfectly aligned detector for proton-proton collisions at 7 TeV in the center of mass. Note that because of the  $p_T^\mu > 25$  GeV/c requirement, the mean  $\langle 1/p_T^\mu \rangle$  is a function of  $\eta$ . At large  $\eta$ , because of electroweak interference, the mean  $\langle 1/p_T^\mu \rangle$  for positive muons (shown in blue) and negative muons (shown in red) is different.

shown in blue and negative muons which are shown in red.

In general, an overall momentum scale (e.g. error in the  $B$  field) should be the same for positive and negative muons. A misalignment would result in a difference in the mean  $\langle 1/p_T^\mu \rangle$  between positive and negative muon. A portion of the muon momentum correction that corrects for a misalignment is additive in  $1/p_T^\mu$ . A portion of the muon momentum correction that corrects an inaccurate  $\int \mathbf{B} \cdot d\mathbf{L}$  is multiplicative in  $1/p_T^\mu$ , and is the same for positive and negative muons.

Therefore, the correction factors  $C^{Data/MC}(Q, \eta, \phi)$  for positive and negative muons are then regrouped to form two different corrections,

- A muon momentum scale multiplicative correction ( $D_m$ ) that could originate from an incorrect integral of  $\mathbf{B} \cdot d\mathbf{L}$ .
- An additive correction for the bias ( $D_a$ ) that could originate from misalignment.

In addition, we define an overall scale correction  $G$  which is determined by the known  $Z$  mass peak position. After the momentum scale corrections, we expect to obtain  $G = 1.0$ . In the equations below we refer to the perfectly aligned resolution smeared MC as  $MC(gen)$ , and  $MC(rec)$  denotes the MC at the reconstructed (misaligned) level.

$$C^{Data/MC}(Q, \eta, \phi) =$$

$$\langle 1/p_T^{MC(gen)}(Q, \eta, \phi) \rangle - \langle 1/p_T^{Data/MC(rec)}(Q, \eta, \phi) \rangle$$

$$D_m(\eta, \phi) = (C^{Data/MC}(+, \eta, \phi) + C^{Data/MC}(-, \eta, \phi))/2$$

$$D_a(\eta, \phi) = (C^{Data/MC}(+, \eta, \phi) - C^{Data/MC}(-, \eta, \phi))/2$$

$$\frac{1}{p_{T,\eta,\phi:corrected}^\pm} = \frac{1}{p_T^\pm} \times M(\eta, \phi) \pm A(\eta, \phi)$$

$$M(\eta, \phi) = 1 + \frac{2D_m(\eta, \phi)}{\langle 1/p_T^+ \rangle + \langle 1/p_T^- \rangle}$$

$$A(\eta, \phi) = D_a(\eta, \phi) - \frac{D_m(\eta, \phi)(\langle 1/p_T^+ \rangle - \langle 1/p_T^- \rangle)}{\langle 1/p_T^+ \rangle + \langle 1/p_T^- \rangle}$$

$$p_{T,scale+\eta,\phi:corrected}^\pm = G \times p_{T,\eta,\phi:corrected}^\pm$$

Here,  $C^{Data/MC}$  is the muon momentum correction factor for the data or reconstructed MC in bins of  $Q$ ,  $\eta$ , and  $\phi$  of the muon (e.g.  $8 \times 8$  matrix in  $\eta$  and  $\phi$  for each muon polarity). This  $\langle 1/p_T^\mu \rangle$  correction corrects for the charge,  $\eta$ , and  $\phi$  dependence of the mis-reconstructed momentum, as well as an overall scale to yield the correct  $Z$  mass.

After the application of the multiplicative and additive corrections, the  $Z$  peak position at the reconstructed level in data and MC is tuned with a multiplicative corrections  $G^{data}$  and  $G^{MC}$  (which are expected to be close to 1.0) to agree with that of the perfectly aligned MC. We chose to define the peak position by fitting the generated spectrum (post FSR) in a narrow  $Z$  mass region (88 to 94 GeV) to a Breit-Wigner function.

In addition, we use the parameters  $\Delta$ , and SF, to make sure that the resolution in the reconstructed Monte Carlo matches the resolution in data. Here,  $\Delta$  and SF, are estimated by comparing the overall  $M_{\mu^+\mu^-}$  mass distributions between data and MC (using a  $\chi^2$  test). These parameters, which are only applied to MC events, are used to tune the width of the MC  $M_{\mu^+\mu^-}$  distribution to match the data. This is done via additional  $p_T$  smearing:

$$\frac{1}{p_{T,additional-smearing}^\pm} = \frac{1}{p_T^\pm} + \Delta \times \mathcal{N}(1, SF),$$

where  $\mathcal{N}(\mu, \sigma)$  is a random normal distribution with a mean of  $\mu$  and an rms of  $\sigma$ .

## 5 Muon Momentum Correction (step 2): further tuning using $\Delta M^Z$

The  $\langle 1/p_T^\mu \rangle$  based corrections fully correct for *all* reconstruction bias in the reconstructed Monte Carlo. The average  $Z$  mass in bins of  $Q$ ,  $\eta$  and  $\phi$  for the reconstructed MC after the corrections is the same as for the perfectly aligned MC.

We form the distributions of  $\frac{\Delta M^Z}{M^Z}$  where  $\Delta M^Z = M^Z(\text{measured}) - M^Z(\text{expected})$  for all  $\mu^+$  and  $\mu^-$  in  $\eta/\phi$  bins. We find that after the  $\langle 1/p_T^\mu \rangle$  based corrections (step

1) are applied, the bias in the measurement between positive and negative muons is removed from both the data and reconstructed MC. However, we find that there is scatter in  $\langle M_{\mu\mu}^Z \rangle$  spectra in the data that is larger than in the reconstructed MC. This scatter originates from a small  $\eta$  and  $\phi$  dependence in the trigger and reconstruction efficiencies in data that are not simulated perfectly in the MC. Mis-modeling of the transverse momentum dependence of the efficiency for different  $\eta$  and  $\phi$  yields an incorrect value of  $\langle 1/p_T^\mu \rangle$ .

We correct for the additional scatter in the data by using the deviation in the average invariant mass  $\Delta M^Z$  of  $\mu\mu$  events in each of the  $\eta/\phi$  bins for  $\mu^+$  and  $\mu^-$  to fine tune the momentum correction.

For each  $\eta/\phi$  bin in the data, the value of  $\Delta M^Z$  can be different from zero if the momentum scale for one of the muons in that bin ( $p_1$ ) is slightly off. The fluctuations in the momentum scale for the other muon leg ( $p_2$ ), which can end up in any place in the detector, averages to zero. This is because after the application of the  $\langle 1/p_T^\mu \rangle$  based corrections, all biases are removed and the average of the momentum scale corrections for a large number of  $\eta$  and  $\phi$  bins is zero.

The relation between  $\frac{\Delta M}{M}$  and  $\frac{\Delta p_\mu}{p_\mu}$  can be extracted from the following expressions:

$$\begin{aligned} M_{Z-Data}^2(Q, \eta, \phi) &= 2p_1p_2(1 - \cos \theta) \\ 2\Delta M \times M_{Z-Data}(Q, \eta, \phi) &= \Delta p_1 \times (2p_2(1 - \cos \theta)) \\ 2\Delta M \times M_{Z-Data}(Q, \eta, \phi) &= \frac{\Delta p_1}{p_1} \times (M_{Z-Data}) \\ 2\frac{\Delta M}{M}(Q, \eta, \phi) &= \frac{\Delta p_1}{p_1} \end{aligned}$$

Therefore, we fine tune the mean  $\langle 1/p_T^\mu \rangle$  based correction by an additional factor of  $1 + 2\frac{\Delta M}{M}(Q, \eta, \phi)$ . We do this iteratively until the distribution for  $\frac{\Delta M^Z}{M^Z}$  has the smallest rms about zero. We refer to this step as the combined  $\langle 1/p_T^\mu \rangle$  and  $\Delta M$  based correction.

In addition to removing bias, we find that for a CMS like detector the momentum resolution for 1 TeV muons is improved from  $\pm 8\%$  before the application of momentum scale/alignment corrections to  $\pm 4\%$  after corrections. In the measurement of the mass of 125 GeV Higgs boson in the four muon channel, the systematic error in the mass scale is reduced from 0.4% before the application of momentum scale/alignment corrections to less than 0.1% after corrections.

## 6 Systematic Errors

Once we fine tune the corrections using the  $\frac{\Delta M}{M}$  distributions, we find that most of the systematic errors are removed. The two step procedure is insensitive to the modeling of the efficiencies, backgrounds, or modeling of the rapidity and  $p_T$  spectrum for the production of  $Z/\gamma^*$  bosons. As a test for the LHC samples, we removed the  $p_T$  tuning from the generated MC sample and did not subtract

any of the backgrounds from data samples. We repeated the entire process, and the resulting coefficients extracted from the combined  $\langle 1/p_T^\mu \rangle$  and  $\Delta M$  based corrections remained unchanged.

The errors in the momentum scale corrections originate primarily from the statistical errors in the  $\mu^+\mu^-$  sample. The samples are of  $\approx 0.5$  million  $\mu^+\mu^-$  events for the Tevatron, and a few million  $\mu^+\mu^-$  events for the LHC, respectively.

## 7 Conclusion

Precision measurements such as the charge asymmetry in the production of  $W$  bosons, the measurement of the forward-backward asymmetry in  $Z/\gamma^* \rightarrow \mu\mu$  events as a function of mass ( $A_{fb}$ ), measurements of angular distributions, and searches for new high mass states decaying to  $\mu^+\mu^-$  pairs are very sensitive to bias in the measurement of muon momenta.

We presented a simple method for the extraction of corrections for bias in the measurement of the momentum of muons in hadron collider experiments. Such a bias can originate from a variety of sources such as detector misalignment, software reconstruction bias, and uncertainties in the magnetic field ( $\int \mathbf{B} \cdot d\mathbf{L}$ ). The corrections are obtained by using the average  $\langle 1/p_T^\mu \rangle$  of muons from  $Z/\gamma^* \rightarrow \mu\mu$  events in bins of charge,  $\eta$ , and  $\phi$  and further tuned using the  $\mu^+\mu^-$  invariant mass distributions.

The  $M_{\mu^+\mu^-}$ ,  $A_{fb}$ , and  $\phi_{CS}$  distributions are used as reference plots to test the procedure. After the application of the combined  $\langle 1/p_T^\mu \rangle$  and  $\Delta M/M$  based muon momentum correction, any reconstruction bias which may in general be a function of charge,  $\eta$ , and  $\phi$  is completely removed. All kinematic distributions which are used as reference plots show good agreement with those expected for a perfectly aligned detector with no reconstruction biases.

## References

1. The coordinates in the laboratory frame are  $(\theta, \phi, z)$ , where  $\theta$  is the polar angle relative to the direction of one of the proton beams (the  $+z$  axis), and  $\phi$  the azimuth. The pseudorapidity is  $\eta = -\ln \tan(\theta/2)$ . For an  $\mu^+\mu^-$  pair  $p_T = P \sin \theta$ ,  $E_T = E \sin \theta$ , the rapidity  $y = \frac{1}{2} \ln \frac{E+p_z}{E-p_z}$ , where  $P$  and  $p_z$  are the magnitude and  $z$  component of the momentum, and  $E$  is the energy of the  $\mu^+ + \mu^-$  pair.
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